Water is ubiquitous and so an understanding of water's anomalous liquid state is crucial for such diverse fields as protein biochemistry, meteorology or astrophysics. A postulated first order phase transition between two distinct one-component liquids at low temperatures is believed to be the key to many riddles in contemporary science: a "fragile" liquid of high density and a "strong" liquid of low density. At higher temperatures the phase boundary might end in a speculative second critical point in supercooled water. Unfortunately it has not been possible so far to support/falsify these hypotheses with direct experiments because of fast crystallization of the liquid(s) in the relevant portion of the phase diagram, which is called "no man's land". Therefore, experiments to test the hypothesis were previously done in the non-crystalline, solid state ("amorphous water") at temperatures well below the "no man's land". More than 20 years ago liquid-like relaxation was measured on heating glassy water at 1 bar to 136 - 150 K, i.e., to temperatures slightly below crystallization, which is still discussed controversially. Recently we managed to observe liquid-like properties on heating high density amorphous ice (HDA) under isobaric conditions at pressures up to 1 GPa above its glass-liquid transition at a temperature slightly below the "no man's land" without observing significant crystallization. These findings open the exciting possibility to characterize deeply supercooled liquid water both at ambient and high pressure conditions and to check if water indeed shows a first order liquid-liquid phase transition between two distinct liquids. With the aid of this volumetric method we can furthermore, for the first time, investigate whether also the other amorphous states of water, e.g., very high density amorphous ice (VHDA), show a glass→liquid transition at high pressures and possibly clear the haze surrounding water's glass transition. In addition to dilatometry we will probe the glass transition using calorimetry and dielectric relaxation at pressures of up to ~2 GPa and study how the molecular structure of equilibrated amorphous ice cooled from the liquid evolves with increasing pressure by isotope substitution neutron diffraction. This will unravel the question how many liquids and how many corresponding amorphous states there are in water, and if VHDA discovered by us in 2001 shows a polyamorphic transition to HDA, or if it is simply annealed HDA.